An electron injected into liquid helium pushes helium atoms apart and forms a small cavity, or "electron bubble". Close to absolute zero, the electron bubble displaces more than 700 atoms, creating a cavity with radius of about R = 2 nm.

a) Consider the wave functions of the trapped electron, assuming that the boundary of the spherical bubble acts as an infinite potential. Show that *spherically symmetric wave functions* have the form $\psi(r) = u(r)/r$, where $u(r)$ obeys a particularly simple differential equation.

b) Determine the functional form of $\psi(r)$ for the ground state and first excited state, and calculate the energies (in eV) for these two states.

c) The probability of finding the electron between r and $r + \Delta r$ is given by $P(r)\Delta r$. Determine P(r) and sketch it for the ground state and first excited state on separate graphs, indicating any zeroes or infinities in these functions.

d) Derive an expression for the equilibrium radius of the bubble, R, in terms of the pressure p on the liquid and fundamental constants, and evaluate R for p = 1 atm. Neglect the surface tension of the liquid, and assume that the temperature is low enough that only the electron ground state is significantly occupied.

e) Derive an approximate formula for the bubble radius using the Heisenberg uncertainty principle.

Useful constants: 1 atm = 10^5 Pa = 10^5 N/m^2. $h^2/2m_e = 1.505$ eV·nm^2