Consider a simple harmonic oscillator in one dimension with the usual Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2$$

(a) The eigenfunction of the ground state can be written as

$$\psi_0(x) = Ne^{-\alpha^2 x^2/2}$$

Determine the constants N and α.

(b) What is the energy eigenvalue of the ground state?

(c) At time $t=0$, an electric field $|E|$ is switched on, adding a perturbation of the form $H_I = e|E|x$. What is the new ground state energy?

(d) Assuming that the field is switched on in a time much shorter than $\frac{1}{\omega}$, what is the probability that the oscillator stays in the ground state immediately afterwards?