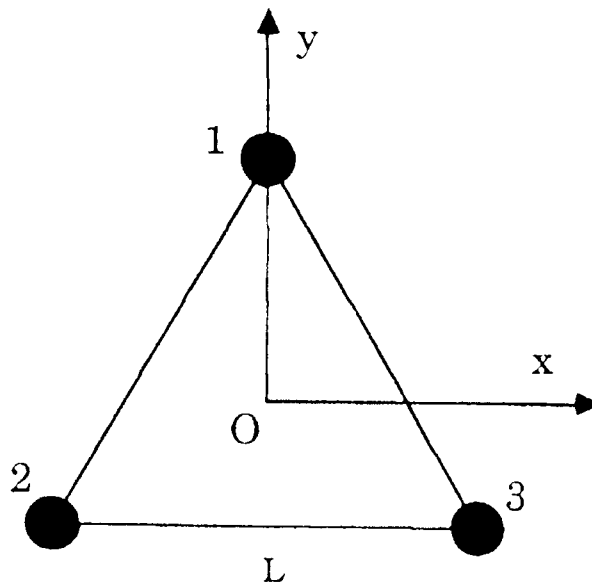


A molecule consists of three equal atoms which form an equilateral triangle of side  $L$  as shown. The eigenstates of an electron in this molecule can be considered as linear combinations of orthonormal basis states,  $\phi_i$ , centered on each atom  $i = 1, 2, 3$ . In this basis the Hamiltonian has diagonal matrix elements  $\langle \phi_i | H | \phi_i \rangle = \varepsilon$ , and off-diagonal matrix elements between neighboring basis states  $i$  and  $j$ ,  $\langle \phi_i | H | \phi_j \rangle = -t$ , where the energy  $t > 0$ .



(a) Define the operator  $R$  which operates on the basis functions as follows:

$R\phi_i = \phi_{i+1}, i = 1, 2; R\phi_3 = \phi_1$ . Note that  $R$  has the effect of rotating the basis functions around the triangle and that  $R^3 = 1$ . Show that  $R$  commutes with the Hamiltonian.

(b) The eigenstates  $\psi_n$  of  $R$  with eigenvalues  $\lambda_n$  can be expressed as  $\psi_n = \sum_{i=1}^3 a_{ni} \phi_i$ .

Find the expansion coefficients  $a_{ni}$  and the eigenvalues  $\lambda_n$ .

[Hint: note that  $\lambda_n^3 = 1$ .]

(c) Give the energy eigenvalues in terms of  $\varepsilon$  and  $t$  and give their degeneracies.

(d) Now suppose an electric field of strength  $F$  is switched on in the  $+x$ -direction.

The only effect is to change the diagonal matrix elements of the Hamiltonian at each site by an amount  $eFx_i$ , where  $x_i$  is the  $x$  position of the center of atom  $i$ , and  $-e$  is the charge of the electron. Find the new energy eigenvalues to linear order in  $F$ .