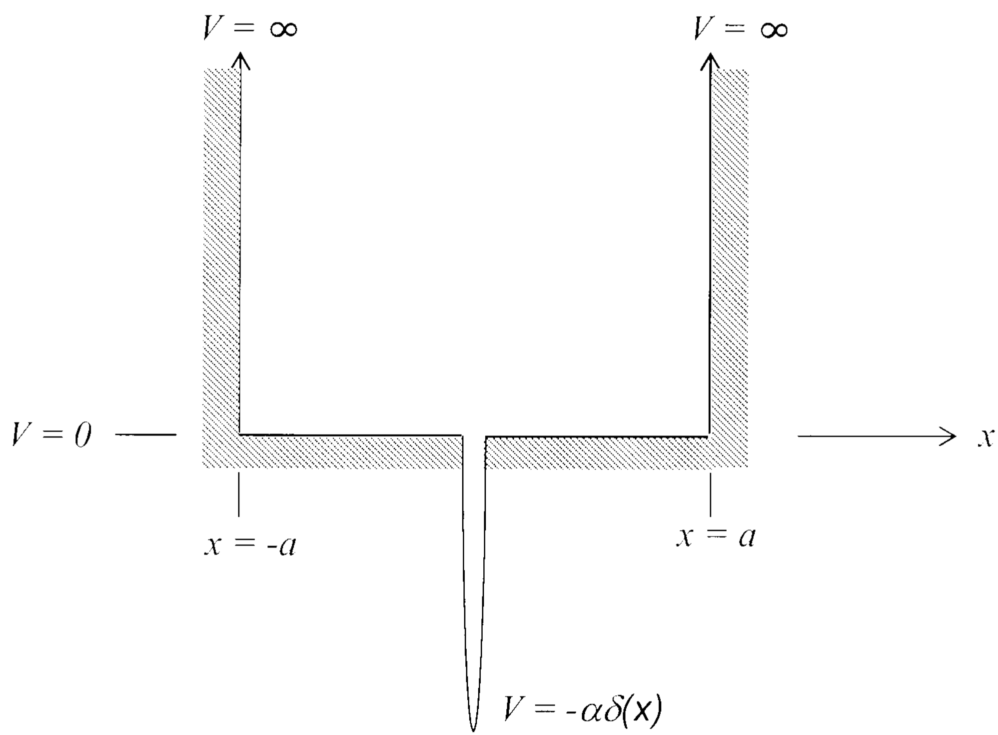
A particle of mass m moves in a one-dimensional potential well having infinitely high potential walls at $x = \pm a$, and an attractive one-dimensional δ-function potential well of strength α located at $x=0$:

$$V(x) = -\alpha\delta(x) \qquad |x| < a$$

$$V(x) = \infty \qquad |x| \geq a$$



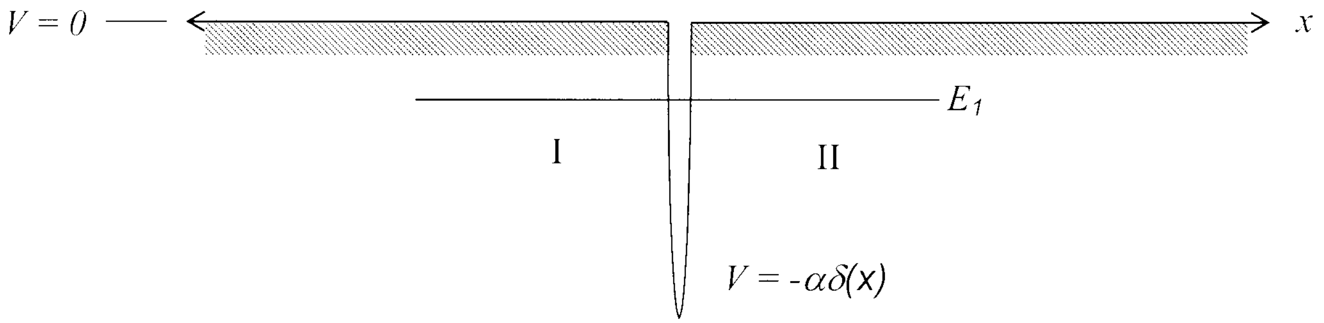
(a) Show by integrating the Schrödinger equation across the δ-function that the wavefunction Ψ for this potential exhibits a discontinuous change of slope across the δ-function given by:

$$d\Psi/dx|_{0+} - d\Psi/dx|_{0-} = -(2m\alpha/\hbar^2)\ \Psi(0)$$

(b) Assuming that α is small, so that there are no bound states in the δ-function potential, qualitatively sketch the wavefunctions associated with (i) the ground state, (ii) the 1st excited state, and (iii) the 2nd excited state of the particle confined in the potential above.

(c) Assume that α is small, so that the δ-function potential can be treated as a small perturbation on the infinite square well potential ($\alpha=0$). For the lowest three energy levels, use first-order perturbation theory to calculate the first-order energy difference $\Delta E_n^{(1)}$ between the eigenvalues of the full potential shown above ($\alpha\neq 0$), and those of the infinite square well potential ($\alpha=0$).

(d) If the infinite potential walls are moved to $x = \pm\infty$ (as shown below), then for a sufficiently large value of α, one can show that there will be one bound state of the δ-function potential with an energy E_1. Sketch this bound state wavefunction of the δ-function potential.



(e) For a particle bound to the δ-function potential above, use the proper wavefunctions in regions I and II, as well as appropriate boundary conditions, to determine the bound state energy E_1.