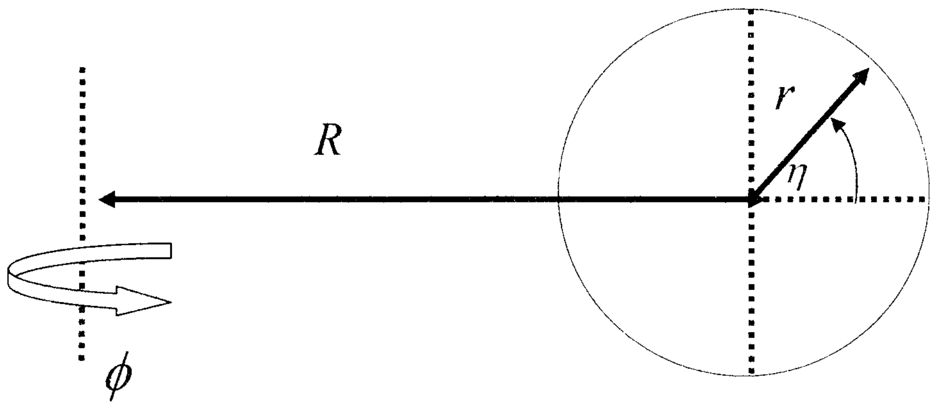


An electron of mass $\mu$ is constrained to lie on a torus (the surface of a donut) with the dimensions depicted above. The torus is the surface that is swept out as the circle is rotated about the indicated axis as $\phi$ is varied from $0 \rightarrow 2\pi$. The potential is zero when the electron is on the torus, and infinite when it is off of the torus. In the limit where $r \ll R$, the kinetic energy $(T)$ of the electron can be written as $T = \frac{1}{2}\mu R^2\dot{\phi}^2 + \frac{1}{2}\mu r^2\dot{\eta}^2$. This implies an approximate Hamiltonian appropriate to the $r \ll R$ limit given by $\hat{H} = \frac{\hat{L}^2}{2\mu R^2} + \frac{\hat{J}^2}{2\mu r^2}$ where $\hat{L} = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$ and $\hat{J} = \frac{\hbar}{i}\frac{\partial}{\partial\eta}$

(a) Write the form for the general wave function for solutions of the Hamiltonian in the $r \ll R$ limit. Carefully define the two integral quantum numbers that describe these solutions.

(b) Among those stationary states with degeneracy of four, compute the energy of state with the lowest energy. Work in the $r \ll R$ limit. Your expression should be in terms of the variables $\mu$, $r$, and $R$ and physical or mathematical constants as required.

(c) Work out an exact expression for the classical kinetic energy. Use this expression to construct the exact Hamiltonian of the electron in terms of $\hat{J}, \hat{L}, \mu, r, R,$ and $\eta$

(d) One can approximate the difference between the exact Hamiltonian and the approximate Hamiltonian by $\Delta H = \hat{L}^2\left(A\ \cos\eta + B\ \cos^2\eta\right)$ where $A$ and $B$ are real constants. Use first order degenerate perturbation theory to compute the splitting of the four degenerate states discussed in part (b) due to this perturbation. Your expression for the energy splitting should be in terms of the variables $A$, $B$ and physical or mathematical constants as required.