Consider a system of two distinguishable particles, each with spin $\frac{1}{2}\hbar$. In this problem all degrees of freedom other than the spins are to be ignored. Let $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$ be the vector operators for spins of the particles. Let the Hamiltonian of this system be

$$\hat{H} = A\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{s}}_2,$$

where A is a constant.

(a) Calculate the energy spectrum of this system.

Now, assuming at $t = 0$, particle 1 has spin up $\left(s_{1,z} = \frac{1}{2}\hbar\right)$ and particle 2 has spin down $\left(s_{2,z} = -\frac{1}{2}\hbar\right)$.

(b) Express the wavefunction of this system at $t = 0$ in terms of the eigenstates of the Hamiltonian.

(c) At $t > 0$, find the probability for particle 1 to have spin up $\left(s_{1,z} = \frac{1}{2}\hbar\right)$.

(d) At $t > 0$, find the probability for the total spin of the system, S, to have a value of $\hbar$.

(e) At $t > 0$, what are the expectation values for $s_{2,z}, s_{2,x}$, and $s_{2,y}$?