Consider the one-dimensional problem of a particle of mass m in a gravitational field g. Let z be the coordinate giving the height above an impenetrable floor.

a) Write down the time-independent Schrödinger equation that describes the particle in an eigenstate of energy E.

b) Give an expression for E_0, the ground-state energy, (in terms of m, g, and Planck's constant $\hbar$) which is correct to within a dimensionless factor.

c) Sketch the ground-state wavefunction, $\psi_0(z)$, and the first excited state wavefunction , $\psi_1(z)$, for both positive and negative z.

d) How many nodes does $\psi_n(z)$, the wavefunction of the n^{th} excited state, have for $z > 0$?

e) Find $\langle\psi_n|z|\psi_n\rangle$ in terms of E_n, m, g, and $\hbar$, again to within a dimensionless factor. Is that dimensionless factor bigger than or smaller than one? (Explain how you know.)

f) Using the results of the previous parts, find an approximate expression for the dependence of E_n/E_0 on n, for $n \gg 1$.