QM 7a1102A

Here are three short problems regarding the quantum mechanics of spin.

Problem 1

(a) A spin-$\frac{1}{2}$ particle is subjected to a homogenous magnetic field along the y-axis at time $t = 0$. State the Hamiltonian of the system in the basis of eigenstates of the spin operator $\hat{S}_z$, i.e., the states $|\frac{1}{2}, \frac{1}{2}\rangle$, $|\frac{1}{2}, -\frac{1}{2}\rangle$ in the $|s, s_z\rangle$ basis.

(b) Determine an initial spin state that does not change, except by an overall phase factor, after the magnetic field is applied.

(c) Assume that the spin-$\frac{1}{2}$ particle considered in (a) is initially polarized along the positive z-axis, i.e., is in the state $|\frac{1}{2}, \frac{1}{2}\rangle$. Determine the probability to find the particle subsequently in state $|\frac{1}{2}, -\frac{1}{2}\rangle$ at time t.

Problem 2

Now consider two spin-$\frac{1}{2}$ particles 1 and 2 that are described through the spin operators $\vec{S}_1$ and $\vec{S}_2$. They are subject to exchange interaction, i.e., their Hamiltonian is $H = J\, \vec{S}_1 \cdot \vec{S}_2$. There is no magnetic field acting on the spins. Determine the stationary states and associated energies of the system.

Problem 3

Finally, consider the decay of a neutron n

$$n \rightarrow p + e + \bar{\nu}\,. \tag{1}$$

Assume that the neutron decays from a spin state $|\frac{1}{2}, \frac{1}{2}\rangle$. Which (normalized) wave function for the spins of p (proton), e (electron), and $\bar{\nu}$ (antineutrino) results from the decay under the condition of conservation of total spin?