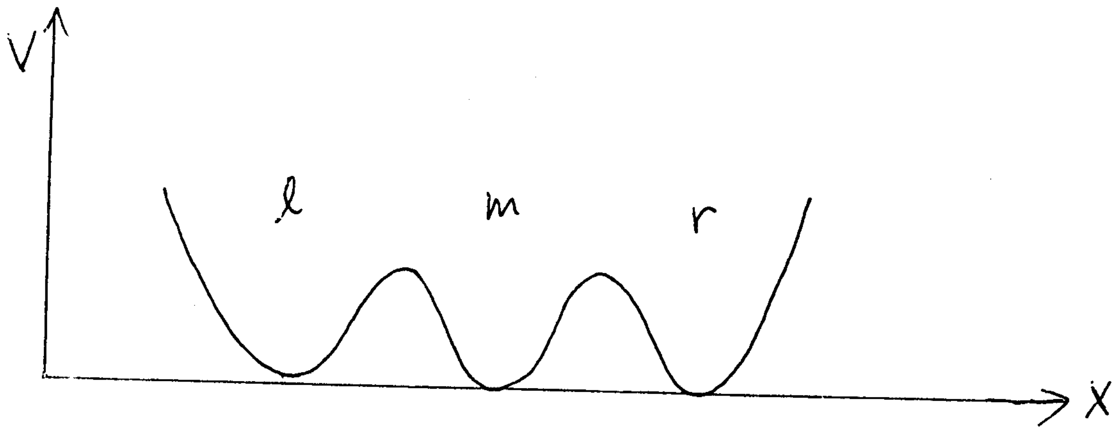
QM Spring 02 A



This problem concerns a model of a particle moving in one dimension in a three well potential, as shown in the figure. The wells are well separated and have the same curvature near their minima.

(a) Copy the figure into your exam booklet. In the approximation where there is no tunneling between the wells, sketch the wave function for the ground state and first excited state in each well.

In the rest of this problem, we reduce the original situation to a three-dimensional hilbert space, spanned by three vectors which represent the ground states in each well. The goal is to understand the way these three states mix via tunneling. Only this three-dimensional hilbert space of states is considered from here on. We define our three basis states as follows:

$$|l> = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ |m> = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ |r> = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

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where l, m, r refer to left, middle, and right wells. On these states, we define a parity operation P as follows:

$$\mathrm{P}|l> = |r>$$
$$\mathrm{P}|m> = |m>$$
$$\mathrm{P}|r> = |l>$$

(b) Write out P as a matrix, with rows and columns labeled as shown.

$$P = \begin{array}{c} \\ l \\ m \\ r \end{array} \begin{array}{c} \begin{array}{ccc} l & m & r \end{array} \\ \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \end{array}$$

(c) Show that $P^2|\Psi>=|\Psi>$, where $|\Psi>$ is any state in our three dimensional space.

(d) Construct a normalized state with negative parity, i.e. one satisfying $P|\Psi>=-|\Psi>$.

(e) There are two positive parity states satisfying $P|\Psi>=|\Psi>$. Construct any two normalized, orthogonal states with positive parity.

In this three-dimensional space, the Hamiltonian is

$$H = \begin{array}{c} \\ l \\ m \\ r \end{array} \begin{array}{c} \begin{array}{ccc} l & m & r \end{array} \\ \begin{pmatrix} E_0 & \Delta & \delta \\ \Delta & E_0 & \Delta \\ \delta & \Delta & E_0 \end{pmatrix} \end{array}$$

(f) Show that parity is a good quantum number, that is, eigenstates of H can also be taken to be eigenstates of P.

(g) Find the eigenvalue of H for the negative parity state.

(h) Find the eigenvalues of H for the positive parity states. Also find the corresponding normalized eigenstates. These will in general be linear combinations of the basis you found in (e).