QM FALL 01 B

This problem deals with Fermions of spin $\frac{1}{2}$ and mass m. The Fermions are confined to a two dimensional square box that has length L in each direction.

a) If there is one Fermion in the box, write down explicitly the normalized wave functions (functions of space and spin) and energies for each of the states with the lowest two distinct energies. Indicate explicitly which states are degenerate in energy.

b) If there are two Fermions in the box, neglecting any interaction, give the wave functions and energies for each of the states with the lowest two distinct energies. Give your solution in terms of the functions defined in part a). State carefully the spin and space parts of the wave functions and indicate explicitly any degeneracies.

Now suppose there is a spin-independent interaction between the particles of strength V_0 and very short range a.

c) What is the symmetry of the spatial part of the ground state wave function under interchange of the two particles?

d) Use perturbation theory to calculate the change in energy of each of the states in part b) to lowest order in the interaction.

e) From the results of part d), is the ground state predicted to have the same symmetry for all values of V_0 (both positive and negative)? In light of your answer to part c), what can you conclude about the results from part d)?

Relation that may be useful: $\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$