QM FALL 01 A

The Hamiltonian of a harmonic oscillator of mass m in one dimension is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

where the frequency, ω, is a constant. The energy of the n^{th} state, $|n\rangle$, is

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \, .$$

Raising and lowering operators

$$\hat{a}_\pm = \frac{1}{\sqrt{2m}}(\hat{p} \pm im\omega\hat{x})$$

may be defined, which have the properties

$$\begin{aligned} \hat{a}_+|n\rangle &= i\sqrt{(n+1)\hbar\omega}\,|n+1\rangle \\ \hat{a}_-|n\rangle &= -i\sqrt{n\hbar\omega}\,|n-1\rangle \, . \end{aligned}$$

a) Calculate the commutator $[\hat{a}_+, \hat{a}_-]$.

b) Consider adding the interaction

$$\hat{H}' = b\hat{x}$$

(where b is a constant) to the Hamiltonian. Show that, at first order in perturbation theory, the energy of the n^{th} state does not change. You may use raising and lowering operators, but are not required to do so.

c) The relativistic correction to the kinetic energy is

$$\hat{H}' = -\frac{\hat{p}^4}{8m^3c^2} \, .$$

Use perturbation theory to calculate the first-order correction to the harmonic oscillator energies. You may use raising and lowering operators, but are not required to do so.