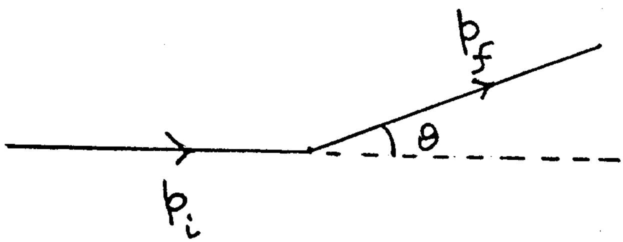
QMSpring01A

(a) A beam of particles of mass m_b and momentum $\mathbf{p}_i$ is scattered by a weak potential H_I that can be considered as a small perturbation: $H = H_0 + H_I$,

$$H_0 = -\frac{\hbar^2}{2m_b}\nabla_b^2 \qquad \text{and} \qquad H_I = \lambda e^{-(r_b/\alpha)^2}$$

where r_b is the position of the particle measured from the scattering center.



(a.1) What is the magnitude $|\mathbf{q}|$ of the momentum transferred by the potential to the scattered particle as a function of $|\mathbf{p}_i|$ and the scattering angle θ.

(a.2) Using the golden rule for transition probability:

$$T_{i\to f} = \frac{2\pi}{\hbar}|\langle f|H_I|i\rangle|^2 \rho_f,$$

where ρ_f is the density of final states, calculate the differential scattering cross section:

$$d\sigma = \frac{T_{i\to f}}{\text{incident flux}}$$

to order λ^2. Your answer may contain $|\mathbf{q}|$.

(b) Now consider the case in which the above beam of particles is incident on a target particle of mass m_t bound in a potential $W(\mathbf{r}_t)$. The Hamiltonian is

$$H_0 = -\frac{\hbar^2}{2m_b}\nabla_b^2 + H_t \quad \text{where} \quad H_t = -\frac{\hbar^2}{2m_t}\nabla_t^2 + W(\mathbf{r}_t)$$

and

$$H_I = \lambda e^{-(\mathbf{r}_b-\mathbf{r}_t)^2/\alpha^2}$$

Let $\phi_0(\mathbf{r}_t)$ and $\phi_1(\mathbf{r}_t)$ be the normalized ground and first excited eigenfunctions of H_t with energies E_0 and E_1. We observe both elastic scattering, and also inelastic scattering where the target is excited to the state $\phi_1(\mathbf{r}_t)$.

(b.1) What are the wave functions of the initial and final states for elastic and inelastic scattering. You may use $\mathbf{p}_f$ to denote the momentum of the scattered particle without calculating it as a function of θ.

(b.2) Calculate the elastic and inelastic scattering cross sections to order λ^2. You may use $\mathbf{q}$ to denote momentum transfer without calculating it as a function of θ.

(b.3) Show that in the limit $|\mathbf{q}| \to 0$ the elastic scattering cross section becomes identical to that for scattering by the potential $\lambda e^{-(r_b/\alpha)^2}$ considered in part (a), while the inelastic cross section becomes zero.

Note: $\int d^3r\, e^{-r^2/\alpha^2}\, e^{-i\mathbf{k}\cdot\mathbf{r}} = \pi^{3/2}\,\alpha^3\, e^{-k^2\alpha^2/4}$.