QMFall00B

Consider a spin - $\frac{1}{2}$ particle with charge e and mass m. The Hamiltonian that describes its interaction with a time-dependent magnetic field $\boldsymbol{B}(t)$ is

$$H_0 = -\frac{ge}{2mc}\boldsymbol{S}(t)\cdot\boldsymbol{B}(t),$$

where g is a constant and $\boldsymbol{S} = (\hbar/2)\boldsymbol{\sigma}$ is the spin operator, in terms of the Pauli operator $\boldsymbol{\sigma}$.

A) Show that the evolution of the spin is described by the equation

$$\frac{d\boldsymbol{S}(t)}{dt} = \frac{ge}{2mc}\,\boldsymbol{S}(t)\times\boldsymbol{B}(t).$$

B) Suppose that the magnetic field is $\boldsymbol{B} = B_0\,\hat{\mathbf{z}}$, where B_0 is a constant and $\hat{\mathbf{z}}$ is the unit vector in the z direction. Draw a diagram showing qualitatively the motion of $\boldsymbol{S}$ relative to $\boldsymbol{B}$.

C) Assume that $\boldsymbol{B}$ is the same as in part B and that the spin is pointing in the x direction at time $t = 0$. What are the expected values of the x, y, and z components of the spin as functions of time for $t > 0$?

D) Suppose that the magnetic field and the initial conditions are the same as in part C, except that now a very brief magnetic field pulse $\boldsymbol{B}_1(t) = \mathrm{B}_0\,\tau_1\,\delta(t-\varepsilon)\,\hat{\mathbf{y}}$ is applied to the particle at time ε, i.e., immediately after $t = 0$ (treat ε as arbitrarily small). Assume that τ_1 is small enough that the effects of the magnetic pulse can be treated to first order. Write down the condition on τ_1 needed to guarantee this (express your answer in terms of e, m, g, and B_0). What are the expected values of the x, y, and z components of the spin as functions of time for $t > \varepsilon$?

You may use the Pauli matrices to solve this problem, if you wish, but they are not necessary. They are $\sigma_x \equiv \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\sigma_y \equiv \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$, and $\sigma_z \equiv \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$.