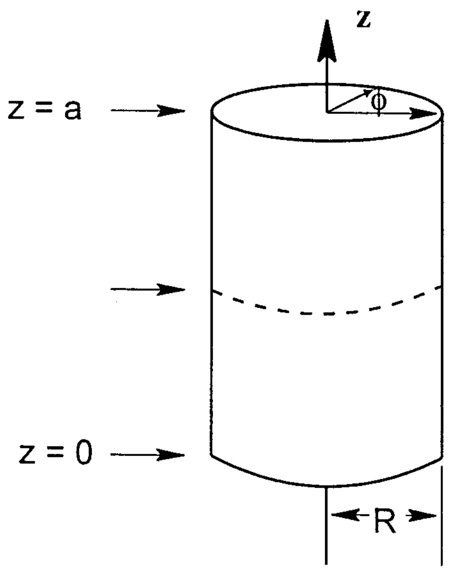
QMFall00A

An electron of mass μ is constrained to lie on a cylinder of radius R and length a as illustrated below:



The electron experiences no potential while on the cylinder and satisfies a time independent Schrödinger equation of the form:

$$-\frac{\hbar^2}{2\mu R^2}\frac{\partial^2\psi}{\partial\phi^2}-\frac{\hbar^2}{2\mu}\frac{\partial^2\psi}{\partial z^2}=E\,\psi \quad \text{while on the cylinder with } 0<z<a \tag{1}$$

A) Use the above Schrödinger equation and boundary conditions $\psi(\phi, z=0)=\psi(\phi, z=a)=0$ to show that stationary solutions of the following form are possible:

$$\psi(\phi, z)\propto \sin Kz\, e^{im\phi} \tag{2}$$

B) Give all allowable values for the quantum number m.

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C) Give all allowable values for K in Eq. (2) in terms of an integer quantum number *(n)* and physical or numerical constants.

D) Obtain an expression for the stationary state energies in terms of the quantum numbers n and m.

E) At time t=0, the electron has the wave function $\psi(\phi, z) \propto \sin Kz \cos^2 \phi$. Calculate both the expectation value and variance ($\sigma^2(L_z) \equiv \langle L^2_z \rangle - \langle L_z \rangle^2$) of L_z of the electron at time t = 0. Will the probability density function for this electron be independent of time? Will the expectation value of L_z be independent of time?

F) When in a particular stationary state, the electron has its current density $\vec{J}$ in the $\hat{\phi}$ direction. $|\vec{J}|$ is maximal at $z = a/4$ and $3a/4$. $\vec{J}$ is zero at $z = 0$, $a/2$, and a. Give a set of m and n quantum numbers for a stationary state that will have the current density which is shown in the figure.