Consider a simple one electron atom without spin orbit coupling interacting with a coherent monochromatic laser beam which is treated classically. Take the interaction Hamiltonian to be $H_{int} = -e\vec{r} \cdot \vec{E}_0 cos\omega t$, where $\vec{E}_0$ is the electric field amplitude of the laser beam, ω is the laser frequency and $\boldsymbol{r}$ is the electron position operator. Take $\vec{E}_0$ to be linearly polarized in the z direction.

(a) Quote the selection rules for the orbital angular momentum and its projection on the z-axis, and electron spin that govern a first order perturbative description of the transitions the light can cause?

Further simplify the problem by considering only two eigenstates of the noninteracting Hamiltonian which satisfy the selection rules, namely the ground state, $u_1(r)exp(-i\omega_1 t)$, and an excited state, $u_2(r)exp(-i\omega_2 t)$. Use the following expression for the interacting atomic wave function:

$$\psi(\vec{r},t) = a(t)u_1(\vec{r})exp(-i\omega_1 t) + b(t)u_2(\vec{r})exp(-i\omega_2 t).$$

At time t=0, the atom is in its ground state, $a = 1$ and $b = 0$.

(b) Derive a pair of differential equations that describe the time evolution of $a(t)$ and $b(t)$, in the form:

$$\dot{a} = A_1 a + B_1 b \quad \text{and} \quad \dot{b} = A_2 a + B_2 b$$

Find explicit expressions for A_1, B_1, A_2 and B_2 in terms of the parameters of the problem.

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Now make the approximation that the light frequency is tuned near to and in the vicinity of the transition frequency, i.e., that $|\omega_2 - \omega_1 - \omega| << \omega$, and ignore terms in your derivation that are too rapidly varying.

(c) For light tuned exactly on resonance, *a* and *b* are periodic functions of time. Find explicit expressions for *a(t)* and *b(t)*. What physical parameters control the frequency of these oscillations?

(d) At what points in time is the atom completely in its excited state, and at what points in time is the atom completely in its ground state? Express your answer in terms of the frequency found in part (c).