Consider the 1s state of the hydrogen atom. The energies of the states in the simplest model of hydrogen are $E = -13.6/n^2$ eV, where n is the principal quantum number. We denote the electron spin by **S**, the electron orbital angular momentum by **L** and the nuclear spin by **I**. The nuclear and electronic spins are coupled by the hyperfine interaction $V_{hf} = A_{hf}\mathbf{I}\bullet\mathbf{S}$, where $A_{hf} > 0$. In this problem we will use the $|F, m_F\rangle$ basis, where $\mathbf{F} = \mathbf{I} + \mathbf{J}$, $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

(a) Determine the 1s energy eigenvalues (relative to the simple ground state above) and their degeneracies, taking into account the hyperfine interaction between the electronic and nuclear spins. Sketch the resulting energy level diagram, and indicate the degeneracies.

We now gradually turn on a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. The Zeeman interaction energy is

$$V_Z = -\mu \bullet \mathbf{B}$$

where μ is the total magnetic moment

$$\mu = \frac{-e}{m_e}\mathbf{S} - \frac{e}{m_e}\mathbf{L} + \frac{eg_p}{2m_p}\mathbf{I}$$

e is the charge of the proton and g_p is its g-factor ($g_p = 5.58$).

(b) State the approximate value (within 10%) of the ratio of the electron spin magnetic moment to that of the proton.

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(c) Neglecting the proton spin contribution to the total magnetic moment μ, determine the 1s eigenstates in the presence of the magnetic field and including the hyperfine interaction, again using the $|F, m_F\rangle$ basis. Which two $|F, m_F\rangle$ eigenstates remain eigenstates in the presence of the magnetic field? Sketch the energies of all the eigenstates as a function of the magnetic field B, and indicate their degeneracy, if any.

(d) At high fields (such that the Zeeman energy V_Z is much larger than the hyperfine energy V_{hf}), are the electron and proton spins effectively coupled together? Explain briefly why or why not.