QMFall99A

An atom of tritium (hydrogen isotope ^3H containing a nucleus $^3H^+$ surrounded by an "atomic" electron) is in its ground state. The $^3H^+$ nucleus suddenly decays into a helium nucleus ($^3He^{2+}$), with the emission of a fast electron which leaves the atom without perturbing the atomic electron.

(a) Find the probability that the resulting $^3He^+$ ion (new nucleus + atomic electron) will be left in a $1s$ state.

(b) Do the same for the $2s$ state.

(c) What is the selection rule for the angular momentum quantum number l in the transition?

HINT: The explicit form of the wave functions for the $1s$ and $2s$ states of a hydrogen-like atom with nuclear charge Z is

$$\psi_{100}^{(Z)}(\vec{r}) = 2\left(\frac{Z}{a_o}\right)^{\frac{3}{2}} \exp\left(\frac{-Zr}{a_o}\right) Y_{00}(\theta,\varphi)$$

$$\psi_{200}^{(Z)}(\vec{r}) = \frac{1}{\sqrt{2}} \left(\frac{Z}{a_o}\right)^{\frac{3}{2}} \left(1 - \frac{Zr}{2a_o}\right) \exp(-Zr/2a_o)\, Y_{00}(\theta,\varphi))$$

where a_o is the Bohr radius and $Y_{00}(\theta,\varphi) = 1/\sqrt{4\pi}$ is the spherical harmonic $Y_{\ell m}(\theta,\varphi)$ for angular momentum zero. The general form of the wave functions of the stationary bound states of the electron in hydrogen-like atoms is

$$\psi_{n\ell m}(\vec{r}) = R_{n\ell}\,(Zr/a_o)\; Y_{\ell m}(\theta,\varphi).$$