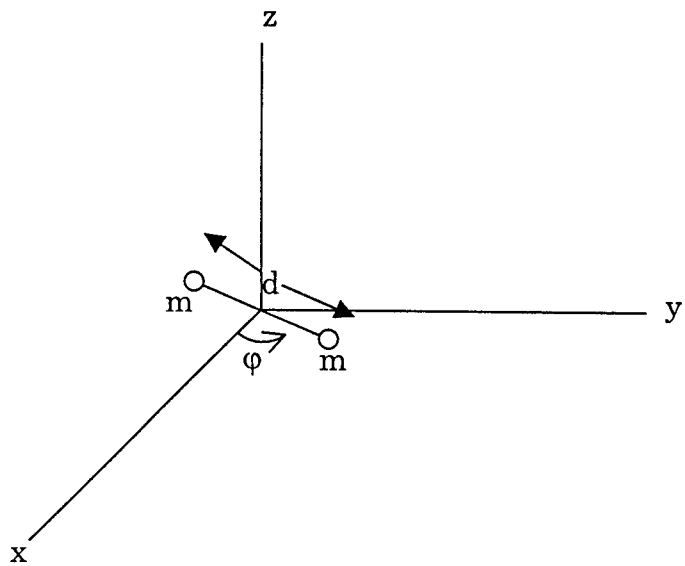
QMSpring99B

Consider a plane rotator: two rigidly connected particles of mass m rotating in the xy plane about their center of mass. The rod of length d connecting the particles has negligible mass.



(a) Write down the system's Hamiltonian in terms of m, d and the angle φ .

(b) Suppose the initial state of the rotator is given by the wave function

$$\psi(\varphi, t = 0) = A \cos^2 \varphi$$

where A is a constant. Consider a single measurement of the angular momentum L_z in this state.

- What are the possible values of L_z observed in that measurement?
- What are the probabilities for each value?
- What is the expectation value of L_z^2 in this state?

(c) Find $\psi(\varphi,t)$ for $t > 0$, given the initial state in part (b). What is the angular frequency ω_1 of the periodic time dependence of $\psi(\varphi,t)$?

(d) Suppose that at $t = t_0$, the distance d between the particles of the plane rotator collapses suddenly to d/2. What is the new Hamiltonian for $t > t_0$? Solve for $\psi(\varphi,t)$ for $t>t_0$.

(e) What is the new angular frequency ω_2 for $t>t_0$? How does the ratio ω_1/ω_2 compare to the same quantity calculated classically?