QM Fall98A

A particle of charge Q, mass M, and zero spin is constrained to move on the surface of a sphere of radius R.

A) Find the energy eigenfunctions, in spherical polar coordinates (θ,ϕ) and the corresponding eigenvalues. Give the appropriate quantum numbers and their allowed range of values. Indicate any degeneracy.

B) Suppose that a uniform magnetic field $\vec{B} = B\hat{z}$ is present. This adds an additional term H_1 to the Hamiltonian of (A),

$$H_1 = -\frac{Q\vec{B}\cdot\vec{L}}{2M},$$

where $\vec{L}$ is the orbital angular momentum of the particle. Find the new eigenvalues of energy and corresponding eigenfunctions. Indicate your results on a diagram of the two lowest energy states found in (A).

C) Instead of a magnetic field, suppose that the system is in its ground state and that at time t=0 we suddenly apply an electric field $E\hat{z}$, which adds an additional term H_2 to the Hamiltonian of (A),

$$H_2 = -Q\,E\,z = -Q\,E\,R\cos\theta .$$

We wish to use first order perturbation theory to find the subsequent behavior of the system, and express the solution to the new Schrödinger equation as

$$\psi(t) = \sum_k D_k(t)\psi_k^{(0)} e^{-iE_k^{(0)}t/\hbar},$$

where $\psi_k^{(0)}, E_k^{(0)}$ are the eigenfunctions and eigenvalues of energy for the unperturbed system found in (A). Here, the symbol "k" refers to the quantum number(s) labeling the states. Show that the coefficients satisfy

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$$\frac{dD_s(t)}{dt} = \frac{1}{i\hbar} \sum_k D_k(t) < s \mid H_2 \mid k > e^{-i(E_k^{(0)} - E_s^{(0)})t/\hbar}$$

D) The electric field is turned off after a time τ which is so short that it is appropriate to assume that $D_k(t) \approx D_k(0)$ on the right-hand side of the above equation. Show that in that case there is only one non-vanishing matrix element in the sum in (C), and thus that only one unperturbed state other than the original ground state will be populated. Calculate the probability that the system will be found in that excited state at $t = \tau$.