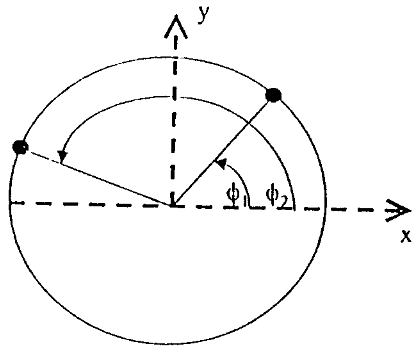
QMSpring98A

Two distinguishable particles with the same mass, m, are constrained to move on a circle of radius R with a mutual repulsion given by

$$V(\phi_1,\phi_2) = V_0 \cos(\phi_1 - \phi_2)$$

where ϕ_1 and ϕ_2 are their respective azimuthal angles and V_0 is a positive constant.



(a) Show that by introducing the two variables

$$\alpha = \phi_1 - \phi_2 \qquad\qquad \beta = (\phi_1 + \phi_2)$$

the solution to the time-independent Schrödinger equation may be written

$$\psi(\phi_1,\phi_2) = u(\alpha)\, v(\beta)$$

Find the differential equations satisfied by $u(\alpha)$ and $v(\beta)$.

(b) Show that the boundary conditions on $\psi(\phi_1,\phi_2)$ imply that $u(\alpha+4\pi) = u(\alpha)$, $v(\beta+2\pi) = v(\beta)$ together with the condition $u(\alpha+2\pi)\, v(\beta+\pi) = u(\alpha)v(\beta)$. Solve the equation satisfied by $v(\beta)$ and find its the eigenvalues.

(c) Explain, qualitatively, why the particles tend to opposite sides of the ring in the limit that V_0 is large and positive.

(d) When V_0 is large and positive, find the energy of zero point oscillations about the equilibrium point $\alpha=\pi$.