QM Fall97B

Consider a system of four distinguishable particles, each with spin 1/2. In this problem all degrees of freedom other than the spins are to be ignored, and $\hbar = 1$. Let $\hat{s}_1$, $\hat{s}_2$, $\hat{s}_3$ and $\hat{s}_4$ be the operators for spins of the particles.

(a) What is the eigenvalue of $\hat{s}_1^2$?

(b) How many states are needed to make a complete orthogonal basis set for this system?

(c) What are the possible eigenvalues of $\hat{S}^2$, where

$$\hat{S} = \hat{s}_1 + \hat{s}_2 + \hat{s}_3 + \hat{s}_4$$

is the operator for the total spin.

(d) Calculate the energy spectrum of this system for the Hamiltonian

$$\hat{H}_0 = (\hat{s}_1 \cdot \hat{s}_3 + \hat{s}_2 \cdot \hat{s}_4)\, A$$

where A is a constant.

(e) In the presence of a magnetic field B in the z-direction, the Hamiltonian becomes:

$$\hat{H} = \hat{H}_0 + gB\,(\hat{s}_{z,1} + \hat{s}_{z,2} + \hat{s}_{z,3} + \hat{s}_{z,4}),$$

where g is a constant and $\hat{s}_{z,i}$ denotes the operator for the z-component of the spin of particle i. Sketch how the energy levels of $\mathbf{H}_0$ will split in a weak magnetic field ignoring terms of order B^2 and higher.