QM 7a//97A

Consider a particle of mass m which is constrained to move on a sphere of radius R in three-dimensions. Denote the angular coordinates by (θ,ϕ).

(a) Write the Hamiltonian for such a system in terms of the angular momentum operator and constants. What are the energy eigenvalues and level degeneracies?

(b) Suppose the particle has electric charge q and suppose there is a uniform electric field $\mathbf{E} = \mathcal{E}\hat{z}$. Write the Hamiltonian for the particle. Using perturbation theory, find the shift in energy to first order in $\mathcal{E}$ for all states.

(c) Now focus on the ground state. To second order in $\mathcal{E}$, how do you expect the energy to change? Demonstrate your answer by computing in second-order perturbation theory the shift in the ground state energy.

(d) The induced electric dipole moment, $\mathbf{P} = \alpha\mathbf{E}$ where α is the polarizability. Calculate α.