QMSpring 97B

Consider the quantum mechanics of an electron with charge -e and mass M moving on a ring of radius R located in the xy plane. The electron moves in the presence of a uniform magnetic field $\vec{B} = B\hat{z}$. The quantum mechanical Hamiltonian of this system is

$$H = \frac{1}{2MR^2}\left(p_\varphi - \frac{e\Phi}{2\pi c}\right)^2$$

where φ is the angular coordinate of the particle on the ring, $p_\varphi \equiv \frac{\hbar}{i}\frac{\partial}{\partial\varphi}$ is the momentum conjugate to φ, $\Phi = B\pi R^2$ is the magnetic flux and c is the speed of light.

(a) What commutation relations do the operators φ and p_φ obey? What boundary conditions do the wave functions of this system satisfy? Justify your answer.

(b) Find the exact eigenstates and the energy levels E_n of this system, giving the physical meaning and the allowed values of the quantum numbers which label the states. Write your answers in terms of the magnetic flux Φ and the flux quantum $\phi_0 = 2\pi\frac{\hbar c}{e}$.

Hint: Write the flux Φ as $\Phi = (k+\alpha)\phi_0$, where k is an integer and α is a real number in the interval $-\frac{1}{2} \le \alpha < \frac{1}{2}$.

(c) Find the ground state of this system for arbitrary values of the magnetic flux Φ. For what values of Φ does the ground state become degenerate? What are the quantum numbers of the degenerate states? Sketch the energy of the ground state as a function of Φ/ϕ_0.