QMSpring97A

The deuteron is a weakly bound state of a neutron and a proton. The nuclear interaction between the proton and neutron can be approximated by a three-dimensional square well potential in the center-of-mass frame:

$$V(r) = \begin{cases} -V_N & r < r_N = 10^{-13}\,\text{cm} \\ 0 & r > r_N \end{cases}$$

where r is the separation between the proton and the neutron at positions $\vec{r}_p$ and $\vec{r}_n$ respectively. In the center-of-mass frame the two-body problem reduces to that of a single particle with reduced mass μ moving in the potential well $V(r)$.

(a) Derive the reduced mass μ in terms of the neutron and proton masses, m_n and m_p and find the radial Schrödinger equation for this system.

(b) Assuming a bound state exists, what is the angular momentum of the ground state? Find the ground state wave function in the center-of-mass frame.

(c) Whether or not the neutron and the proton form a bound state depends on V_N and r_N. Determine V_{min}, the minimum value of V_N for a bound state to exist, in terms of r_N and μ. Calculate V_{min} in MeV using $m_p = 938\ \text{MeV}/c^2$ and $m_n = 940\ \text{MeV}/c^2$. (Note: $\hbar c = 1.9732710^{-11}\text{MeV cm}$)

(d) Suppose that V_N is changed by a fraction δ: $V_N = V_{min}\,(1 + \delta)$. Determine the ground state energy to leading order in δ when $0 < \delta << 1$.