QM7a1196A

Consider the ground state of a collection of identical non-interacting bosonic atoms of mass m in an isotropic three dimensional harmonic oscillator well centered on the origin. (This model is a reasonable starting point for understanding recent experiments on cooling of spin-polarized atoms in magnetic traps). Let ω be the angular frequency of the well. Consider first the properties of a single particle in the well.

(a) If the spread in the single particle ground state wave function in space is of order R, estimate the potential energy and kinetic energy of the particle. Then estimate the value of R by minimizing the total energy.

(b) Write down the Hamiltonian and Schrödinger equation for a particle in the well. Let $\vec{r}$ be the position coordinate.

(c) Write down the normalized ground state wave function of a particle in the well, and the ground state energy.

(d) Compute the mean radius squared $\langle r^2 \rangle$ of a particle in the well, and verify that it is consistent with the result in (a).

Now suppose that there are N particles in the ground state in the well (a Bose condensate).

(e) Give the normalized ground state wave function of the N particles.

(f) Calculate the local number density $\rho(r)$ of particles.

Problem 3 con't.

(g) Now assume that the particles interact via a very short ranged repulsive potential which produces an interaction energy per particle $\upsilon\rho(\mathrm{r})$, where υ is a positive constant. If the cloud of particles has size of order R estimate the interaction energy per particle.

(h) Using your results from (a) for the kinetic energy and potential energy per particle and (g) for the interaction energy of a cloud of size of order R, estimate the size of the cloud of particles in the limit of large N.

Show that for large N the ground state for large numbers of particles is determined by a balance between the attractive harmonic oscillator force of the well and the repulsive interaction between the particles, and therefore the kinetic energy is smaller than the potential energy by a factor $\sim N^{-4/5}$.