QM Spring 96A

(a) The process of "K-capture" involves the capture of an inner orbital electron by the nucleus, resulting in a reduction of the nuclear charge by one unit. This process is possible in part due to the non-zero probability that the electron can be found within the volume of the nucleus. An electron is in the 1s state of a hydrogenic potential, with wave function given by

$$\Psi(r) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/a_o} , \qquad (1)$$

where Z is the atomic number and a_o is the Bohr radius; calculate the probability that a 1s electron will be found within the nucleus. Take the nuclear radius to be $R = 10^{-5}$ Å, and assume that the wave function $\Psi(r)$ can be approximated by $\Psi(0)$ for $r < R$, because the nuclear radius is much smaller than the Bohr radius, $R << a_o$.

(b) Assume that an electron is initially in the hydrogenic ground state described in Eq. (1) with $Z = 2$. A nuclear reaction abruptly changes the nuclear charge to $Z = 1$. What is the probability that the electron will be found in the ground state of the new potential after the change in nuclear charge?

(c) Assume that instead of the final state described in part (b), the nuclear reaction leaves the electron in a state given by

$$\Psi(r,\theta,\varphi) = A\,(\sin\theta \sin\varphi + \sin\theta \cos\varphi + \cos\theta)\, r e^{-r/a_o} ,$$

where A is a constant such that $|\Psi|^2$ is normalized to unity. What are the possible values that can be obtained in measurements of L^2 and L_z, and with what probabilities will these values be measured?