QM Fall 1995 B

A non-relativistic electron of mass m can move in only one dimension and its wave function $\psi(x)$ is described by the following time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi(x) = E\psi(x).$$

This question concerns tunnelling through narrow potential barriers, which will be approximated by delta functions.

(a) Consider one delta-function barrier at $x = 0$, described by $V(x) = V_0\delta(x)$. Show that the equations which relate the wave function and its first derivative on left (L) and right (R) sides of the potential barrier have the following form, and give an expression for A:

$$\psi_R(0) = \psi_L(0) = \psi(0),$$

$$\psi_R'(0) - \psi_L'(0) = A\psi(0).$$

(b) A beam of electrons of mass m is incident on a delta-function potential at $x = 0$, and the wave function on the left (L) and right (R) sides is written as

$$\psi_L(x) = \exp(ikx) + a\exp(-ikx),$$

$$\psi_R(x) = b\exp(ikx).$$

Which way is the beam traveling? What is the speed of the electrons?

(c) Give an expression for the transmission coefficient $T = |b|^2$ as a function of A and k.

(d) Now consider a situation where there are two delta-function barriers of equal magnitude, but opposite sign, separated by a distance $L = 2\pi/k$:

$$V(x) = V_0(x) - V_0\delta(x-L).$$

What is the value of T?