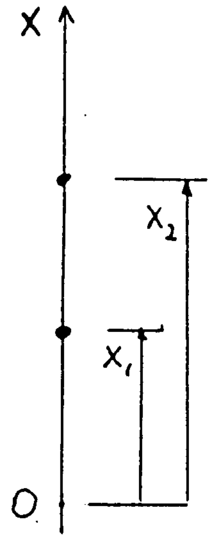
QM Fall95A



Two particles of mass m move in one dimension with coordinates x_1, x_2, and interact with a potential energy

$$V(x_2-x_1) = \frac{1}{2}\, m\omega^2(x_2-x_1)^2.$$

(a) By means of a coordinate rotation from x_1, x_2 to

$$y = (x_2-x_1)/\sqrt{2}; \qquad z = (x_1+x_2)/\sqrt{2}$$

show that the Hamiltonian can be written in the form

$$H(y,z) = H_{ho}(y) + H_{pw}(z),$$

in which H_{ho} has harmonic oscillator solutions $\phi_n(y)$ and H_{pw} has plane wave solutions $\psi_q(z)$. Express the eigenfunction $\Psi_{nq}(y,z)$ for the system in terms of $\phi_n(y)$ and $\psi_q(z)$ and give the corresponding energies E_{nq} in terms of m,ω, and the quantum numbers n and q. Nowhere in this problem do you need to give explicit forms for the harmonic oscillator functions ϕ_n.

[Hint: For a coordinate rotation $\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.]

(b) Given that the rms value of the coordinate for a harmonic oscillator with mass M, frequency Ω, is $\sqrt{(\hbar/2M\Omega)}$, determine $\langle y^2\rangle^{1/2}$ for the ground state of the present problem.

(c) Suppose that the two particles are spinless bosons. By considering how this constrains the solutions, write down the two lowest energy solutions for q = 0, together with their energies. Sketch these eigenstates.

(d) Now suppose instead that the two particles are spin 1/2 fermions, and that appropriate wave functions consist of a product of the above coordinate functions with a spin function. The latter is a combination of the up and down spin functions $s_u(1)$, $s_d(1)$, $s_u(2)$, $s_d(2)$ for the separate particles. By considering how this constrains the solutions, write down explicitly the four lowest energy wave functions, including spin dependence, for q = 0, together with their energies.