BSM. Consider a lattice model of a gas consisting of N identical atoms of type A and N identical atoms of type B. Some of the atoms may be bound into molecules AB with binding energy E (*i.e.* E units of energy are needed to separate the molecule into its atomic components). The lattice has N sites each of which is allowed to contain no more than one of each particle type: A, B or AB. In other words only the following sets of particles

$$\{\text{empty}\}, \quad \{\text{A}\}, \quad \{\text{B}\}, \quad \{\text{AB}\}, \quad \{\text{A}, \text{B}\}, \quad \{\text{A}, \text{AB}\}, \quad \{\text{B}, \text{AB}\}, \quad \{\text{A}, \text{B}, \text{AB}\}.$$

are allowed at each site.

a) Compute the number of microstates for the situation in which the internal energy U of the system is given by $U(r) = -NrE$ — *i.e.* when there are rN molecules AB, $(1-r)N$ unbound atoms of type A, and $(1-r)N$ unbound atoms of type B. In this part of the problem make **no** approximations.

b) Use your answer to part (a) to compute the entropy $S(U)$ for the system when $U(r) = -NrE$. Use Stirling's approximation $\ln(n!) = n\ln(n) - n$ to simplify your answer. When N is large, show that the entropy per site $s \equiv S/N$ becomes independent of N.

c) Use your answer to parts (a) and the quantity s you found in part (b) to write down the free energy $F(T, r)$, where T is the temperature.

d) Explain how to use $F(T, r)$ to compute the equilibrium value of r as a function of T. Find a closed-form expression for $r(T)$ and evaluate it approximately for the case $k_\text{B}T = E/3$. If you have no calculator, you may approximate $e = 2.7182\ldots$ as 3.0.

e) What value will r take when $T = 0$? What value will r take when $T = \infty$? You will get credit for a *physical* explanation of your answers to this part even if you have been unable to do any other parts of the question.