SM

A spacecraft is in orbit around the Sun at distance $R_\oplus = 1.5 \times 10^{11}$m. It is shielded from the Sun's heat by a flat panel that is oriented perpendicular to the Sun and absorbs all of the incoming solar radiation. The Sun can be regarded a black body with surface temperature $T_\odot = 6000$ K. The radius of the Sun is $R_\odot = 7 \times 10^8$ m. The Stephan-Boltzmann constant is $\sigma = 5.6 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$.

a) Derive the formula giving the solar energy flux (the power per unit area) arriving at the panel. Numerically evaluate your formula to give the energy flux in units of Wm^{-2}.

b) The shielding panel is thermally insulated so that it only loses heat *via* photons re-radiated from its front surface. Assuming that the panel can be treated as a black body, calculate the equilibrium temperature of the panel.

c) The free energy of a volume V of black-body radiation is $F(V,T) = \gamma T^4 V$, where γ is some constant that depends on $k_\mathbf{B}$, $\hbar$ and the speed of light c (you do not need to compute it). Use a thermodynamic relation to express the internal energy U, the entropy S, and the pressure P of the volume of gas in terms of γ, V, and T. Hence find the dimensionless constant ζ such that $P = \zeta U/V$.

d) Assuming that the incoming energy flux (the answer to part (a)) is Q_in, compute P_in, the force per unit area due to the impact of the photons on the panel. Compare your answer to the energy density in the incoming radiation. Does the constant ζ from part (c) still apply? If not why not?

e) The photons re-radiated from the front side of the panel exert an additional pressure P_recoil on the panel. Show that $P_\text{recoil} = \kappa Q_\text{out}/c$ where Q_out is the outgoing energy flux and κ is a dimensionless constant that you should find.

Hint: For part (e) observe that the outgoing energy-flux can be written

$$Q_\text{out} = I \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta,$$

where I is the angle-independent *radiance* of the outgoing radiation (power per unit area, per unit solid angle). Here θ is the angle away from the normal to the surface. Understand why the $\cos\theta$ is present and modify this integral to obtain photon momentum-flux.