SM

A model of a rubber band consists of a one-dimensional chain containing a large number N of linked rigid segments as shown in figure 1. Each segment is independent of the others. It occupies one of two possible states: *horizontal*, which contributes length a to the chain, or *vertical*, which contributes nothing to the length. The segments are linked so that they cannot come apart. The chain is in contact with a heat bath at temperature T.

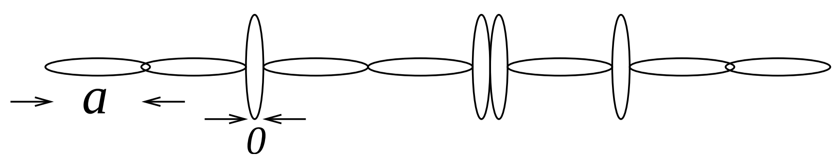

Figure 1: Part of chain.

a) If there is no energy difference between the the two states, what (expressed in terms of Na, and T) is the average length of the chain?

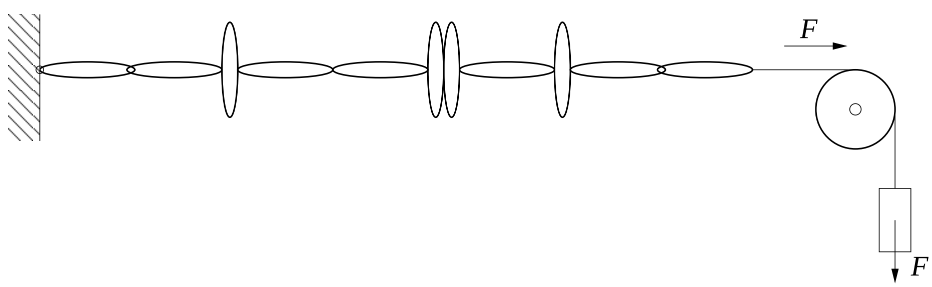

Figure 2. The chain being stretched by force F.

b) The chain is now fixed at one end and stretched horizontally at the other end by a weight hung over a pulley that supplies a horizontal force F. (See figure 2.) Determine the average length of the chain (again in terms of Na, and T) at any temperature. Find the average length in the limits that $T \to 0$ and $T \to \infty$.

c) In which temperature limit does Hooke's Law (that the change in length from equilibrium is proportional to F) apply?

d) As the temperature is raised do we need more or less force to stretch the rubber band? If you warm up the band while applying a fixed force, will it expand or contract? Give a qualitative explanation of the behavior by considering the length dependence of the Helmholtz free energy.