

SMB The free energy of an ideal gas of atoms is

$$F(T, V) = nk_B T (\ln(N/n_Q V) - 1),$$

where  $N$  is the number of atoms,  $V$  the volume of the container,  $T$  the temperature and  $k_B$  is Boltzmann's constant. The quantity  $n_Q$  is the *quantum density* that can be expressed as  $3.03/\lambda^3$  where  $\lambda$  is the de Broglie wavelength of a particle with average thermal energy at temperature  $T$ .

- a) Compute  $n_Q$  for a gas of Ar atoms (atomic weight = 40) at  $T = 300$  K.
- b) Compute the entropy of 1 liter of Ar gas at a pressure of 1 atm and temperature 300 K.
- c) The walls of the container have a total of  $M$  sites at which the Ar atoms can bind with binding energy  $\Delta$ . Show that the free energy of  $N_s$  atoms bound to the surface is

$$F_{\text{bound}} = -N_s \Delta + N_s k_B T (\ln(N_s/M) - 1),$$

assuming that  $M \gg N_s \gg 1$ .

- d) Calculate the binding energy  $\Delta$  in terms of  $N$ ,  $N_s$ ,  $M$ ,  $n_Q$ ,  $T$ ,  $V$ , assuming that  $N \gg M \gg N_s \gg 1$ .
- e) Evaluate  $\Delta$  (preferably in units of eV) assuming that  $V = 1$  liter,  $T = 300$  K and that 1% of the surface sites are occupied.

Useful constants:

$$\begin{aligned} m_p &= 1.673 \times 10^{-27} \text{ kg}, \\ h &= 6.625 \times 10^{-34} \text{ J.s} = 4.140 \times 10^{-15} \text{ eV.s} \\ k_B &= 1.381 \times 10^{-23} \text{ J/K} = 8.6175 \times 10^{-5} \text{ eV/K} \\ 1 \text{ atm} &= 10^5 \text{ Pa}. \end{aligned}$$