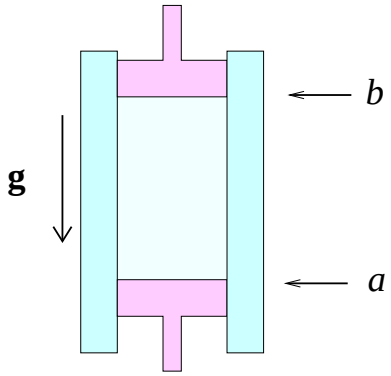
SMA. An ideal Boltzmann gas consists of N weakly interacting monatomic atoms of mass m. It occupies a cylinder of cross-sectional area A whose bottom is closed by a movable piston at $z = a$ and its top is closed by a movable piston at $z = b$.



The gas is in thermal equilibrium at a uniform temperature T, and is immersed in a downward directed uniform gravitational field $\mathbf{g}$. You may assume that the mean-free path of the atoms is short enough that the gas is in equilibrium at each height z.

a) Write down the canonical partition function for the gas in terms of an explicit integral.

b) Evaluate the integral from part (a), and so obtain the free energy $F(T, a, b)$.

c) From the answer to part (b) use a thermodynamic relation to find the entropy S of the gas. You may leave your answer as a derivative of an explicit function of T, A, a, b, $|\mathbf{g}|$, N, m and $\hbar$.

d) Again from your result in part (b), use a thermodynamic relation to find the pressure of the gas at both $z = a$ and at $z = b$.

e) Is your answer for the difference in pressure between $z = a$ and $z = b$ consistent with what you expect from hydrostatic equilibrium of the gas? If not, explain.

Useful formula:

$$\int_{-\infty}^{\infty} \exp\{-ax^2\}\, dx = \sqrt{\frac{\pi}{a}}$$