Q4 Consider an ensemble consisting of a large number N of distinguishable carbon nanotubes all of the same length L and diameter $D \ll L$. Assume that there is exactly one free electron per nanotube.

a) Treat each nanotube as a one-dimensional infinite square well of length L. Write down the expression for the partition function of the electron in a single nanotube. You may leave your expression as a (possibly infinite) sum. You should, for the moment, ignore electron spin.

b) Use the small-T limit of your expression for part (a) to calculate the temperature dependence of the internal energy $U(T)$ and the specific heat $c(T)$ of the nanotube near $T = 0$.

c) We have so far treated the a nanotube as one-dimensional. Make an estimate (expressed in terms of m_e, $\hbar$. L and D) of the temperature above which this is no longer a good approximation.

Now consider the collection of spins of the N electrons as a thermodynamic ensemble. Each spin has magnetic moment μ.

d) A magnetic field B is applied. Ignoring spin-spin and spin-orbit interactions, compute the internal energy $U_{\text{spin}}(B, T)$ of the N spins, and sketch *versus* T. Then calculate the heat capacity $C_{\text{spin}}(B, T)$ and briefly discuss its behavior at both high and low temperatures.

e) A very large B field is now used to force every spin into its lowest energy state. The field direction is then suddenly reversed, so that each spin is in its excited state. What is the change of entropy caused by this sudden reversal? Starting from the all-spins-excited configuration, what is the change of entropy if one (we don't know which) spin flips so as to align with the field.

f) Make a simple estimate of the effective temperature of the resulting single-spin-flipped state.

g) Now some small coupling is switched on between the system consisting of the (single-spin-flipped) spin degrees of freedom and the system consisting of the electron motion at some finite temperature T. Will heat flow from the spins to the electron motion or *vice-versa*? Explain why. (Hint: Will the entropy increase if energy is transferred from the higher temperature system to the lower temperature system?)