Q4 Consider a model for n identical xenon atoms of mass m that are trapped on the surface of a solid. A xenon atom can be tightly bound to one of N adsorption sites with binding energy E_a (a positive number), or it may be free to move over the two-dimensional surface whose total area is A. When it is free to move along the surface, the xenon atom has both kinetic energy $mv^2/2$ and constant potential energy V. You may assume that the number n of xenon atoms is much smaller than the number of adsorption sites N, and that each adsorption site can bind at most one atom.

a) Consider first the two-dimensional gas composed of the n_g atoms that are free to move. Evaluate the partition function of the gas as a function of n_g, A, m, $\hbar$, k_B and the temperature T. Also compute the free energy $F_g = E - TS$ and the chemical potential

$$\mu_g = \frac{\partial F_g}{\partial n_g}$$

as a function of the gas density n_g/A. (You may assume that the temperature is high enough that you may use Boltzmann statistics.)

b) Now consider the $n_b = n - n_g$ atoms that are bound to the adsorption sites. Find the mean energy, entropy, free-energy and chemical potential for these particles.

c) Show that in equilibrium the number n_g is determined by $n_b + n_g = n$ together with

$$\frac{n_b A}{n_g N} = C(T) \exp\left\{\frac{E_a + V}{k_B T}\right\}.$$

You should express the function $C(T)$ in terms of T, m, $\hbar$ and k_B.

Useful formulæ:

$$\begin{aligned} \ln n! &= n \ln n - n, \\ \int_{-\infty}^{\infty} e^{-ax^2} &= \sqrt{\frac{\pi}{a}}. \end{aligned}$$