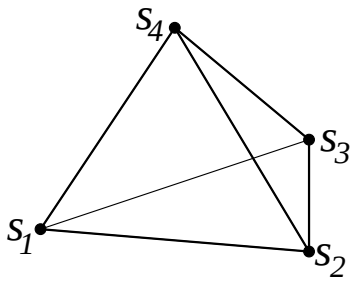
SM Consider four Ising spins (*i.e.* particles with only one degree of freedom) described by variables s_1, s_2, s_3, s_4, whose possible values are $s_j = \pm 1$, $j = 1, \ldots, 4$. The four spins are placed on the vertices of a regular tetrahedron and are subject to an interaction with an external magnetic field h, and in addition a ferromagnetic exchange interaction $-Ks_js_k$, $(K \geq 0)$ across each of the six edges $< j, k >$ of the tetrahedron. (See figure.)



The energy of the system is therefore

$$E[s_1, s_2, s_3, s_4] = -h \sum_{i=1}^{4} s_i - K \sum_{<j,k>} s_j s_k.$$

The system is in contact with a heat bath at temperature T.

a) Write down the partition function Z_0 for the case that the particles do not interact, *i.e.*, for $K = 0$.

b) Now write down the partition function Z for the general case. Collect all identical terms to obtain for Z the most compact expression. What is the value of Z at $T \to \infty$? Explain.

c) From Z evaluate the thermal average magnetization per spin

$$M = \left\langle \frac{1}{4} \sum_{i=1}^{4} s_i \right\rangle.$$

In the case $K = 0$ and $k_{\mathrm{B}}T = 0.25$ sketch the h-dependence of M. What is M for $T \to \infty$?

d) From Z evaluate the specific heat. For $h = 0$ and $K = 1$ sketch the temperature dependence of the specific heat.