4

Consider a paramagnetic material consisting of a volume V of N non-interacting spin-1 particles (with magnetic dipole moment $\vec{\mu} = \frac{\mu}{\hbar}\vec{S}, S_z = m\hbar, m = 0, \pm 1$) in a magnetic field $\vec{B} = B_0\hat{z}$, at temperature T.

(a) Write the partition function in terms of k_BT and $\varepsilon = \mu B_0$.

(b) 1. Calculate the average energy E of the N spins, and from this the average magnetization density M (magnetic moment per unit volume), defined by $E=-MBV$.

2. Find the approximate functional form of M in the low- and high-temperature limits.

3. Sketch M versus ε / kT and briefly give a physical interpretation of your results.

(c) Calculate the isothermal susceptibility $\chi \equiv dM / dB$, and give the approximate functional form in the low- and high-temperature limits.

(d) Now suppose that these spins reside in (and are in thermal equilibrium with) a crystal lattice, which is in good thermal contact with a gas at some non-zero temperature $T=T_1$. We then apply a strong B field of magnitude B_1. Calculate the spin contribution to the entropy, and show that it is only a function of $\mu B / k_BT$.

(e) Now we remove the gas, thereby thermally isolating the spin-lattice system (the spins remain in thermal contact with the crystal lattice). Finally, we turn down the magnetic field strength to a value B_2. Calculate the final temperature T_2 of the spins (and therefore the temperature of the crystal lattice).