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In a cold atom experiment, ^6Li atoms were trapped and the tendency for ferromagnetism was measured. As a simplified description of this experiment consider the following model: Let there be $N = N_\uparrow + N_\downarrow$ non-interacting spin ½ atoms of mass m in a cube with side L and volume $V=L^3$. Here $N_\uparrow$ denotes the number of spin up $(S_z = +½)$ atoms and $N_\downarrow$ the number of spin down $(S_z = -½)$ atoms.

(a) Using periodic boundary conditions, what are the normalized single-particle energy eigenstates and eigenvalues?

(b) In terms of $N_\uparrow$, $N_\downarrow$, V, and m, what is the Fermi wavevector (k_F) and Fermi energy (E_F) of the up and down atoms and what is the total non-interacting ground state energy, E_0? Assume N and L are sufficiently large that one can use integrals instead of sums.

Using a laser-induced interaction, a repulsive contact potential between each pair of atoms is turned on. The total interaction energy is then given by:

$$V_{\text{int}} = \sum_{i<j}^{N} g\delta\left(\vec{r}_i - \vec{r}_j\right)$$

with $g>0$. Here $\vec{r}_i$ is the position of the i^{th} atom.

(c) In first order perturbation theory with respect to g, what is the interaction energy between two atoms that have the same value of S_z?

(d) In first order perturbation theory with respect to g, what is the interaction energy between two atoms that have different values of S_z?

(e) In first order perturbation theory with respect to g, and given $N_\uparrow$ and $N_\downarrow$, calculate the total interaction energy.

(f) Using the non-interacting energy from (b) and the interaction energy from (e), what is the minimum value of g for which the polarized state $(N_\downarrow = 0, N_\uparrow = N)$ is more stable than the unpolarized state $(N_\downarrow = N_\uparrow)$ at zero temperature, $(T = 0)$? Call this the critical $T = 0$ coupling, $g_c(T = 0)$.

(g) Again, considering only the polarized and unpolarized states, determine how the critical coupling $g_c(T)$ depends on temperature in the limit $k_B T << E_F$. Use the expression for the low temperature entropy of a free Fermi gas restricted to a single spin state:

$$S = \frac{\pi^2 N k_B T}{2E_F}$$

and expand the total energy found in (f) to first order in $[g - g_c(T = 0)]$.