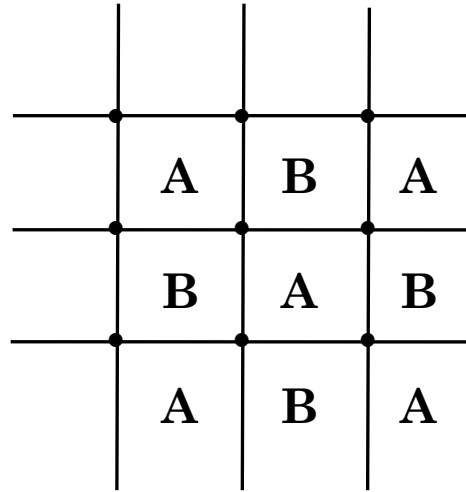


6

N identical classical particles occupy a square lattice with $2N$ sites, with at most one particle per site. Alternate sites are labeled A and B, as sketched below. Denote by c the fraction of particles on the A sites.



- (a) For fixed c , and assuming that all configurations at fixed c are equally likely, (a mean field approximation) calculate the entropy $S(c)$ of the system for large values of N .
- (b) When two objects are on neighboring A and B sites, there is a repulsive interaction energy E_0 . For fixed c , and assuming that all configurations at fixed c are equally likely, show that the average total energy of the system is

$$E(c) = 4NE_0 c(1-c).$$

In thermal equilibrium at temperature T , c is determined by minimizing the free energy $F(c) = E(c) - TS(c)$. This system exhibits a second order phase transition at a temperature T_c .

- (c) Describe the state of the system at very high temperatures. What is the observed value of c ?
- (d) Describe the state of the system at very low temperatures. What are the possible values of c ?
- (e) Determine T_c .

[HINT: For large values of N we can approximate $\ln N!$ by $N \ln N - N$.

For small x , $\ln(1 + x) \approx x - x^2/2 + x^3/3 + \dots$.]