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Consider a very thin wire of length L, made of a superconducting material. The wire remains in the normal metal state for temperatures $T > T_c$, and is superconducting for $T < T_c$, where T_c is the critical temperature and T is the actual temperature of the wire. In the superconducting state (i.e. at $T < T_c$), the excitations above the ground state, called bogoliubons, have a dispersion relation $E_B(p) = \sqrt{E(p)^2 + \Delta^2}$. Here Δ is the superconducting energy gap, and $E(p) = p^2/2m - E_F$ is the spectrum of the normal electronic excitations, occuring in the normal (i.e. nonsuperconducting) state. In this expression p is the momentum, m is the electrons mass and E_F is the Fermi energy.

Within this problem make the following simplifying assumptions: Assume that the gap Δ is constant for $T < T_c$ and zero for $T > T_c$. Assume that the gap is much smaller than the Fermi energy E_F. Assume that all excitations (either normal or bogoliubons) have their momentum $p > p_F$, where p_F is the Fermi momentum. Neglect spins of bogoliubons and normal electrons (which are both fermions). In other words, assume that each momentum state can accept no more than one bogoliubon (in the superconducting state) or no more than one normal electron (in the normal state). Assume that the wire is so thin that the momentum of the excitations is always directed along the axis of the wire, i.e. the wire is one-dimensional.

(a) For temperatures $T > T_c$, at which the wire is normal, determine the density of states in energy D_n. Express the answer in terms of the Fermi velocity of the normal electrons v_F.

(b) At $T = 0$, when the wire is superconducting, calculate the ratio of the density of states in energy of bogoliubons and the normal electrons D_B / D_n. Express the answer in terms of E_B and Δ. [Note that D_n does not depend on energy (E or E_B). The density of states D_n is a constant only depending upon which type of material is used to make the wire.]

(c) Find the ratio of the group velocity for bogoliubons v_B and the Fermi velocity v_F. Express the result in terms of $E(p)$ and E_B.

(d) Assume now a finite temperature of the sample, such that $\Delta / k_B T = 10$. Calculate the average value of the speed of all bogoliubons $\langle v_B \rangle$ excited in the wire due to thermal activation. Assume that $v_F = 10^6$m/s and the density of normal electrons per unit energy in the wire is $D = 10^{25}\text{J}^{-1}$. Also assume that $k_B T = 10^{-23}\text{J}$.