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Consider a collection of non-interacting classical spins. Each spin can take only two values ± 1 in the z-direction. In a magnetic field B in the z-direction the total energy E of this spin system is

$$E = -\mu(N_+ - N_-)B,$$

where μ is the magnetic moment of the spin, N_+ is the number of up-spins, and N_- that of down-spins.

a) Calculate the temperature T of this spin system in equilibrium in terms of $N_\pm$.

b) An electromagnetic pulse is applied that can flip all the spins so that the magnetization reverses its sign without changing its magnitude. The static magnetic field B is maintained as before. Is the obtained state a maximum entropy state or a minimum entropy state? What is the temperature of the spin system after the reversal?

c) An experimenter measures the temperature of the spin system obtained in (b), using an ideal gas thermometer, comprising a 10^{-7} moles of a monatomic non-relativistic ideal gas held at constant volume (you may ignore any internal degree of freedom for this part). The thermometer measures the temperature of the spin system by attaining thermal equilibrium with the spins, a process that affects both the gas and the spins. The spin system contains 0.1 moles of spins. In the magnetic field B, the energy difference between the up and down spin states is 600 K. The initial temperature of the spin system is -500 K (note the sign is not a misprint!). What is the temperature of the spin system after it has come into thermal equilibrium with the ideal gas thermometer? Ignore the possible effect of ionization.

d) Suppose the experimenter infers the temperature obtained in (c) to be 5×10^7K. If the ideal gas is helium, it can ionize with the ionization potentials 24.6 eV and 54.4 eV (note that 50 eV roughly corresponds to 6×10^5 K). If we take into account the possibility of ionization, how must the inferred temperature of the spin system be revised? (Ignore relativistic effects, even if any.)