Consider a crystal made up of N *non-interacting* spin-1 atoms, each of which has a magnetic moment μ. The crystal is placed in a uniform magnetic field of magnitude H, so that each atom has three possible energies:

$E = -\mu H$, when the magnetic moment of the spin is parallel to the field direction;
$E = 0$, when the magnetic moment of the spin is perpendicular to the field direction;
$E = \mu H$ when the magnetic moment of the spin is antiparallel to the field direction.

The crystal has a temperature T.

(a) Without performing detailed calculations, write down values of the internal energy U, the entropy σ, and the net magnetic moment M for this system for each of the following two limiting conditions:

(i) the temperature of the crystal $T \rightarrow 0$

(ii) the temperature of the crystal $T \rightarrow \infty$.

Please provide a physical explanation for your answers.

(b) Calculate the Helmholtz free energy F for this system.

(c) Find an expression for the temperature- and field-dependence of the net magnetic moment, M, of the system in the direction of the applied field, H.

(d) Starting with your result in part (c), write expressions for the asymptotic functional form (up to leading order in the temperature-dependent term) of the average value of the net magnetic moment M/N in the following regimes:

(i) at low temperatures, $k_B T << \mu H$

(ii) at high temperatures, $k_B T >> \mu H$

Verify that your answers reduce to the results you obtained in part (a) under the appropriate limiting conditions.