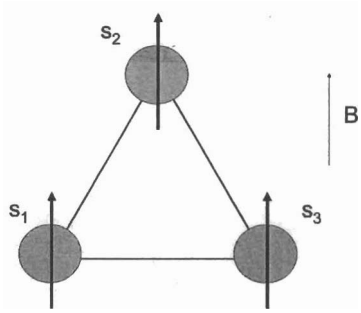

Consider three interacting spins arranged in an equilateral triangle as shown. The z component of each spin s_i $(i = 1, 2, 3)$ can take values $s_i = \pm 1$. There is a magnetic field B in the z direction acting on each spin so that the Hamiltonian is given by

$$H = -J(s_1 s_2 + s_1 s_3 + s_2 s_3) + \mu_B B \sum_{i=1}^{3} s_i,$$

where μ_B is the Bohr magneton and J is the interaction constant.

(a) Give explicit expressions for:
  (1) the partition function at temperature T
  (2) the expectation value $\langle S \rangle$ of the total spin $S = \sum_{i=1}^{3} s_i$
  (3) the expectation value of the term in the energy involving J.

(b) Give the expected form of the spin susceptibility $d\langle S \rangle / dB$ as a function of temperature T for large T.

(c) Show that the result in (b) follows from the expression for $\langle S \rangle$ derived in part (a), by formal calculation.

(d) Copy the figure shown below into your exam book. Plot on this graph in your exam book the zero-temperature phase diagram of the spin system as a function of J and B. Note that the diagram includes both $J > 0$ and $J < 0$. Sketch the configurations of the spins in each region of the phase diagram.