Consider a dilute ideal gas of N spin ½ atoms that have zero orbital angular momentum and magnetic moment, μ. The number density, n, is low enough that Maxwell-Boltzmann statistics apply. The atoms are confined in a volume, V, and held at a temperature, T.

(a) In the absence of a magnetic field, is the heat capacity any different from that of the same number of *spinless* atoms contained in the same volume? Briefly explain your answer. Derive an expression for the heat capacity, $C_V(T)$, of the gas if it is held at constant volume.

(b) Now suppose that there is a non-zero magnetic field, $\vec{B} = B_0\hat{z}$. Derive an expression for $C_V(T,B_0)$ including all degrees of freedom. Evaluate the low field and high field limits of this expression. Using physical reasoning, briefly explain the low field and high field limiting behavior of C_V.

(c) Sketch a curve showing how C_V depends on B_0 for a temperature, T. On the same graph, sketch a curve showing how C_V depends on B_0 for a temperature, $2T$. Clearly indicate the positions of any characteristic features in these curves. Using physical reasoning explain the differences between these two curves.

(d) When B_0 is not zero, derive an expression for C_p, the heat capacity at constant pressure. Using physical reasoning, explain the difference between C_V and C_p.