SM Spring 07 B

N identical classical particles occupy a square lattice with 2N sites, with at most one particle per site. Alternate sites are labeled A and B, as sketched below. Denote by c the fraction of particles on the A sites.

A	В	A
В	A	В
A	В	A

- (a) For fixed c, and assuming that all configurations at fixed c are equally likely, (a mean field approximation) calculate the entropy S(c) of the system for large values of N. Evaluate the entropy when $c = \frac{1}{2}$.
- (b) When two objects are on neighboring A and B sites, there is a repulsive interaction energy E_0 . For fixed c, and assuming that all configurations at fixed c are equally likely, show that the average total energy of the system is $E(c) = 4NE_0 \ c(1-c)$.

In thermal equilibrium at temperature T, c is determined by minimizing the free energy F(c) = E(c)-TS(c). This system exhibits a second order phase transition at a temperature T_c .

- (c) Describe the state of the system at very high temperatures. What is the observed value of c?
- (d) Describe the state of the system at very low temperatures. What are the possible values of c?
- (e) Determine T_c.

HINT: For large values of N we can approximate lnN! by NlnN-N.

For small
$$x$$
, $\ln(1+x) \approx x - x^2/2 + x^3/3 + \dots$