The internal energy U and entropy S of a perfect classical monotomic gas at temperature T are, respectively:

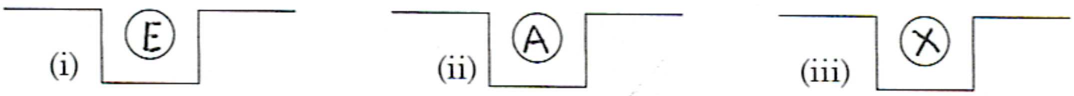
$$U = 3Nk_BT/2; \qquad S = Nk_B[\ln n_o/n + 5/2].$$

Here k_B is Boltzmann's constant, N is the number of atoms, the average concentration is $n = N/V$ and V is the volume, and $n_o = (Mk_BT/2\pi\hbar^2)^{3/2}$, with M the atomic mass.

(a) Show that the chemical potential μ of the gas is related to the pressure p of the gas by

$\mu = -k_BT \ln (n_ok_BT/p)$.

Two such gases, A and X, are in equilibrium with surface sites at which the gases bind to a metal surface. In the presence of A and X simultaneously there are just three possible configurations of each surface site: (i) the surface site is empty (denoted by E below); (ii) the surface site is occupied by one A atom, with energy E_A relative to a stationary A atom in the gas; (iii) the surface site is occupied by one X atom, with energy E_X relative to a stationary X atom in the gas. Excited configurations at the site are not bound, and multiple occupancy is forbidden. You are now asked to consider one site in equilibrium with gases A and X simultaneously at temperature T.



(b) In terms of parameters defined above, write down the partition function Z for the configurations of one site.

(c) Calculate the average concentrations n_E, n_A, n_X for the three configurations of the site in equilibrium with the two gases at temperature T.

(d) At room temperature and fixed partial gas pressures p_A and p_X, the sites are occupied in the ratio $n_E:n_A:n_X = 1:1:18$. Find the maximum value of n_A that can be obtained by varying p_X alone.