Consider a paramagnetic material consisting of a volume V of N non-interacting spin-1 particles (with magnetic dipole moment $\vec{\mu} = \frac{\mu}{\hbar}\vec{S}$, $S_z = m\hbar, m = 0, \pm 1$) in a magnetic field $\vec{B} = B_0\hat{z}$, at temperature T.

(a) Write the partition function in terms of $k_B T$ and $\varepsilon = \mu B_0$.

(b)

1. Calculate the average energy E of the N spins, and from this the average magnetization density M (magnetic moment per unit volume), defined by $E = -MBV$.
2. Find the approximate functional form of M in the low- and high-temperature limits.
3. Sketch M versus ε/kT and briefly give a physical interpretation of your results.

(c) Calculate the isothermal susceptibility $\chi \equiv dM/dB$, and give the approximate functional form in the low- and high-temperature limits.

(d) Now suppose that these spins reside in (and are in thermal equilibrium with) a crystal lattice, which is in good thermal contact with a gas at some non-zero temperature $T = T_1$. We then apply a strong B-field of magnitude B_1. Calculate the spin contribution to the entropy, and show that it is only a function of $\mu B/k_B T$.

(e) Now we remove the gas, thereby thermally isolating the spin-lattice system (the spins remain in thermal contact with the crystal lattice). Finally, we turn down the magnetic field strength to a value B_2. Calculate the final temperature T_2 of the spins (and therefore the temperature of the crystal lattice).