(a) Consider a spinless quantum mechanical particle of mass m in a simple harmonic oscillator potential in one dimension of frequency ω, and at temperature T. The energy of the eigenstate $|n\rangle$ is $E_n = \hbar\omega(n + 1/2)$. Calculate the **partition function**, Z_1, of the particle in the oscillator, and from this result calculate its **free energy**, its **entropy**, and its **mean energy** as functions of the temperature.

(b) Consider two identical spinless Bose particles put in such an oscillator.

(i) What are the possible energy eigenstates and energies of the two bosons in the oscillator. Express the answer in terms of integers, n_1 and n_2, specifying the energy eigenstates of single particles [part (a)].

Determine the degeneracy of each of the lowest four energy levels.

(ii) Calculate the partition function of the two particles at temperature T.

(c) Consider two identical Fermi particles, of spin 1/2, put in such an oscillator, one with spin up and the other with spin down. What is the partition function of the two particles at temperature T?

(d) Now suppose that the two fermions are both in spin-up states in the oscillator.

(i) What are the possible energy eigenstates and energies of the two fermions in the oscillator. Express the answer in terms of integers, n_1 and n_2, specifying the energy eigenstates of single particles. Determine the degeneracy of the lowest four energy levels.

(ii) Calculate the partition function of the two fermions in the oscillator.

(e) Compare the entropy of the two fermions in part (d) at temperature T to that of the two bosons in part (b) at the same temperature. [Hint: You could extract the entropy difference of the two systems from the ratio of their partition functions.] Explain your result in terms of the enumeration of the energy eigenstates in parts (b-i) and (d-i).