Consider a thin coaxial cable of length L. We consider only TEM modes inside the cable, so that the photons may be considered as particles moving in one dimension, with two polarization modes and with a dispersion relation $e = pc$. Here e is the photon energy, p is the momentum of the photon and $c = 3\times10^{8}$ m/s is the speed of light inside the cable. The temperature of the cable is T and the photons are in thermal equilibrium. The ends of the cable are terminated by plugs with zero resistance, such that the electric field is zero at the ends. Neglect zero point energy in all calculations.

(a) Find the allowed wavelengths l_n of the photons that can exist inside the cable (n is the mode number, with n=1 corresponding to the lowest energy mode). Find also the corresponding wave-vectors k_n, frequencies w_n, and energies e_n. Find the lowest possible energy $e_{\min}$ which a photon inside the cable can have. Express the answers in terms of c, Planck's constant ?, and the cable length L.

(b) Calculate the average energy $\bar{e}_n$ as a function of temperature, for a single mode with frequency w_n. Calculate the average number of photons $\bar{m}_n$ in the mode with frequency w_n. Express the answers in terms of Boltzmann's constant k_B, T, ?, and w_n.

(c) Find the energy level spacing Δe for the photons in the cable. State the condition on T and L under which the direct sum over the states can be replaced by a suitable integral, or, in other words, the discreteness of the energy spectrum can be neglected and the spectrum of photons can be considered continuous. Write down the density of states $n(e)$ for the photons inside the cable in such a continuous approximation.

(d) In the continuous density of states approximation, write down the total average energy $\bar{E}$ of the electromagnetic field inside the cable. Give a numerical estimate of $\bar{E}$ for $T = 300K$ and L=3m. In this part only you may use the approximation that $e_{\min} = 0$.

[Hint: $\int_0^\infty \frac{xdx}{e^x - 1} = p^2/6$]

(e) For the case $\Delta e << k_B T$ give an approximate formula for the average number $\bar{N}_>$ of photons in the cable with energies greater than $k_B T$. Numerically estimate $\bar{N}_>$ when $T = 300K$ and L=3m.

(f) For the case $\Delta e << k_B T$ show that the average number $\bar{N}_<$ of photons in the cable with energy between $e_{\min}$ and $k_B T$ is of the form $\bar{N}_< = A\ln(k_B T / e_{\min})$ and provide an expression for A in terms of $e_{\min}$, $k_B T$. Hence estimate the numerical value of $\bar{N}_<$, for $T = 300K$ and L=3m.