A thermally insulated box is divided into two equal compartments of volume V each. In the initial state one compartment is filled with N ideal gas atoms of mass m each at temperature T_i while the other has vacuum. The final state is obtained by pulling out the divider between the compartments without doing any work on the gas. Assume that the interactions between atoms give negligible energy but produce thermal equilibrium over long time, and N and V are both large. Several non-relativistic cases are to be considered. You may ignore energetic contributions of internal degrees of freedom, if any.

Case I: The gas atoms obey classical mechanics.
(a) What are the internal energy E_i and pressure p_i of the initial gas?
(b) What are the internal energy E_f, temperature T_f, and pressure p_f of the final gas after thermal equilibrium?

Case II: The gas atoms are indistinguishable, spin 0, bosons obeying quantum mechanics and T_i is non-zero and small enough for Bose condensation to occur.
(a) Obtain the equation to find the number of atoms N_i^0 initially in the condensate, in terms of N, m, V and T. The integral in this equation need not be evaluated.
(b) Is $T_f < T_i$, $T_f = T_i$ or $T_f > T_i$ and is $N_f^0 < N_i^0$, $N_f^0 = N_i^0$ or $N_f^0 > N_i^0$? Give brief (few line) arguments to justify your answer.

Case III: The atoms are indistinguishable spin $1/2$ fermions and $T_i = 0$.
(a) What are their initial internal energy E_i and pressure p_i?
(b) Is $T_f < T_i$, $T_f = T_i$ or $T_f > T_i$? Give brief arguments.
(c) What is p_f?